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A review of research on laminar mixed convection flow over backward- and forward-facing steps

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Abstract

This paper presents a comprehensive review of the flow and heat transfer results of single-phase laminar mixed convection flow over vertical, horizontal and inclined backward- and forward-facing steps that have been reported in several studies in the open literature. The purpose of this paper is to give a detailed summary of the effect of several parameters such as step height, Reynolds number, Prandtl number, inclination angle, expansion ratio, temperature difference between the heated wall and the free stream, and buoyancy force (assisting and opposing) on the flow and thermal fields downstream of the step. Several correlation equations that were reported in many of these studies to predict the reattachment lengths of the recirculation regions that may develop upstream and/or downstream of the step are also summarized in this review.

1. Introduction

The existence of flow separation and subsequent reattachment due to a sudden expansion or compression in the flow passages, such as backward-facing and forward-facing steps, respectively, play an important role in the design of a wide variety of engineering applications where heating or cooling is required. These heat transfer applications appear in cooling systems for electronic equipment, combustion chambers, chemical processes and energy systems equipment, environmental control systems, high performance heat exchangers, and cooling passages in turbine blades. A great deal of mixing of high and low energy fluid occurs in the reattached flow region of these devices, thus affecting their heat transfer performance. Because of this fact, the problem of laminar and turbulent flow over backward-facing and forward-facing step geometries in forced, natural, and mixed convection have been investigated rather extensively, both numerically and experimentally. However, the current review will only consider the studies of the case of steady-state laminar mixed convection flow over backward- and forward-facing steps. These studies reported a large number of important and interesting experimental and numerical results concerning the flow and heat transfer characteristics of laminar mixed convection flow over backward-facing and forward-facing step geometries. In addition, the Aerospace Heat Transfer Committee (K-12) of the Heat Transfer Division of the ASME held a technical session for a benchmark heat transfer problem at the 1993 ASME Winter Annual Meeting (see Blackwell and Armaly [1,2]). The benchmark problem was to solve a steady-state two-dimensional mixed convection flow of a laminar Newtonian fluid in a vertical channel with a backward-facing step. Eleven papers were contributed in which the benchmark problem was solved numerically.

In the backward-facing step flow geometry, only one separated region develops downstream of the step. On the other hand, in the forward-facing step flow geometry, the flow field is more complicated and one or two separated regions may develop with one upstream and the other downstream from the step, depending on the ratio of the approaching flow boundary-layer thickness to the height of the forward-facing step at the step.

The objective of this paper is twofold. First, is to present a comprehensive review of the flow and heat transfer results of recent studies of single-phase laminar mixed convection flow over backward-facing and forward-facing steps, including some results of the author. Results of interest, such as local heat transfer rate and reattachment lengths and the effects of buoyancy force (assisting and opposing), Prandtl number, step height, inclination angle, Reynolds number, and temperature difference between the heated wall and the free stream on these parameters are summarized and presented. In those studies, a large number of correlation equations were also developed to predict the reattachment lengths of the recir-

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Nomenclature

g	gravitational acceleration	$\mathrm{m{\cdot}s^{-2}}$
Gr_s	local Grashof number, $= g\beta(T_\mathrm{w} - T_\infty)s^3/\nu^2$	
Gr_s^*	modified local Grashof number, $= g\beta q_\mathrm{w} s^4/k\nu^2$	
h	local heat transfer coefficient, $= -k(\partial T/\partial y)_{y=0}/(T_\mathrm{w} - T_\infty)$	$\mathrm{W{\cdot}m^{-2}{\cdot}{}^\circ C^{-1}}$
H	channel height at exit	m
k	thermal conductivity	$\mathrm{W{\cdot}m^{-1}{\cdot}{}^\circ C^{-1}}$
L_j	length of injection port	m
l_j	dimensionless length of injection port, $= L_j/s$	
Nu_s	local Nusselt number, $= hs/k$	
p	pressure	Pa
q_w	local surface heat flux	$\mathrm{W{\cdot}m^{-2}}$
Pe_s	Peclet number,	
Pr	Prandtl number, $= \nu/\alpha$	
R	expansion ratio, $= H/s$	
Re_s	Reynolds number, $= u_\mathrm{o}s/\nu$	
s	step height	m
T	fluid temperature	°C
T_∞	free stream temperature	°C
T_w	wall temperature	°C
u	streamwise velocity component	$\mathrm{m{\cdot}s^{-1}}$
u_o	inlet velocity	$\mathrm{m{\cdot}s^{-1}}$
u_∞	free stream velocity	$\mathrm{m{\cdot}s^{-1}}$
U	dimensionless streamwise velocity, $= u/u_\mathrm{o}$	
v	transverse velocity component	$\mathrm{m{\cdot}s^{-1}}$
v_j	injection velocity	$\mathrm{m{\cdot}s^{-1}}$
V_j	dimensionless injection velocity, $= v_j/u_\mathrm{o}$	
V	dimensionless transverse velocity, $= v/u_\mathrm{o}$	
x, y	streamwise and transverse coordinates	m
X, Y	dimensionless streamwise and transverse coordinates, $= x/s,\ y/s$	
x_e	downstream heated length	m
x_i	inlet length upstream of the step	m
x_r	reattachment length of the recirculation region downstream of the step	m
x_s	length of the recirculation region upstream of the step	m
$X_\mathrm{e}, X_\mathrm{i}, X_\mathrm{r}, X_\mathrm{s}$	$= x_\mathrm{e}/s, = x_\mathrm{i}/s, = x_\mathrm{r}/s, = x_\mathrm{s}/s$	
Greek symbols		
α	thermal diffusivity	$\mathrm{m^2{\cdot}s^{-1}}$
β	coefficient of thermal expansion	$\mathrm{K^{-1}}$
ΔT	temperature difference, $= T_\mathrm{w} - T_\infty$	°C
δ_s	boundary-layer thickness at the step, $= 5x_\mathrm{i}/Re_{x_\mathrm{i}}^{1/2}$	m
θ	dimensionless temperature, $= (T - T_\infty)/(T_\mathrm{w} - T_\infty)$ or $= (T - T_\infty)/(q_\mathrm{w}s/k)$	
ϕ	inclination angle as measured from the vertical axis	deg
ν	kinematic viscosity	$\mathrm{m^2{\cdot}s^{-1}}$
ρ	density	$\mathrm{kg{\cdot}m^{-3}}$
ξ	buoyancy parameter, $Gr_\mathrm{s}/Re_\mathrm{s}^2$ or $Gr_\mathrm{s}^*/Re_\mathrm{s}^2$	

culation regions that develop downstream of the step in the case of a backward-facing step and upstream and/or downstream of the step in the case of a forward-facing step. These correlation equations are also summarized and discussed in this paper. Second, several areas of research that could lead to improvements in our ability to understand and predict this phenomenon will be suggested.

2. Flow geometry

The scope of this review is the research studies that examined single-phase laminar mixed convection flow over two-dimensional backward-facing and forward-facing steps, shown schematically in Fig. 1. In those studies, three different orientations were examined: vertical, horizontal, and inclined. Both uniform wall heat flux and uniform wall temperature heating conditions were considered.

In the experimental studies, the investigators employed a cold-wire constant-current anemometer with boundary layer wire probe and a two-component laser-Doppler velocimeter to measure, respectively, the fluid temperature and velocity distributions at any desired location. Both the backward-facing and forward-facing step geometries along with the physical boundary conditions were modeled for numerical simulations. The numerical studies used the Boussinesq approximation. The resulting non-dimensional governing conservation equations are:

$$\partial U/\partial X + \partial V/\partial Y = 0 \tag{1}$$

$$\begin{aligned} &U\partial U/\partial X + V\partial U/\partial Y \\ &\quad = -\partial P/\partial X + (1/Re_\mathrm{s})\bigl(\partial^2 U/\partial X^2 + \partial^2 U/\partial Y^2\bigr) \\ &\qquad + \bigl(Gr_x/Re_\mathrm{s}^2\bigr)\theta \end{aligned} \tag{2}$$

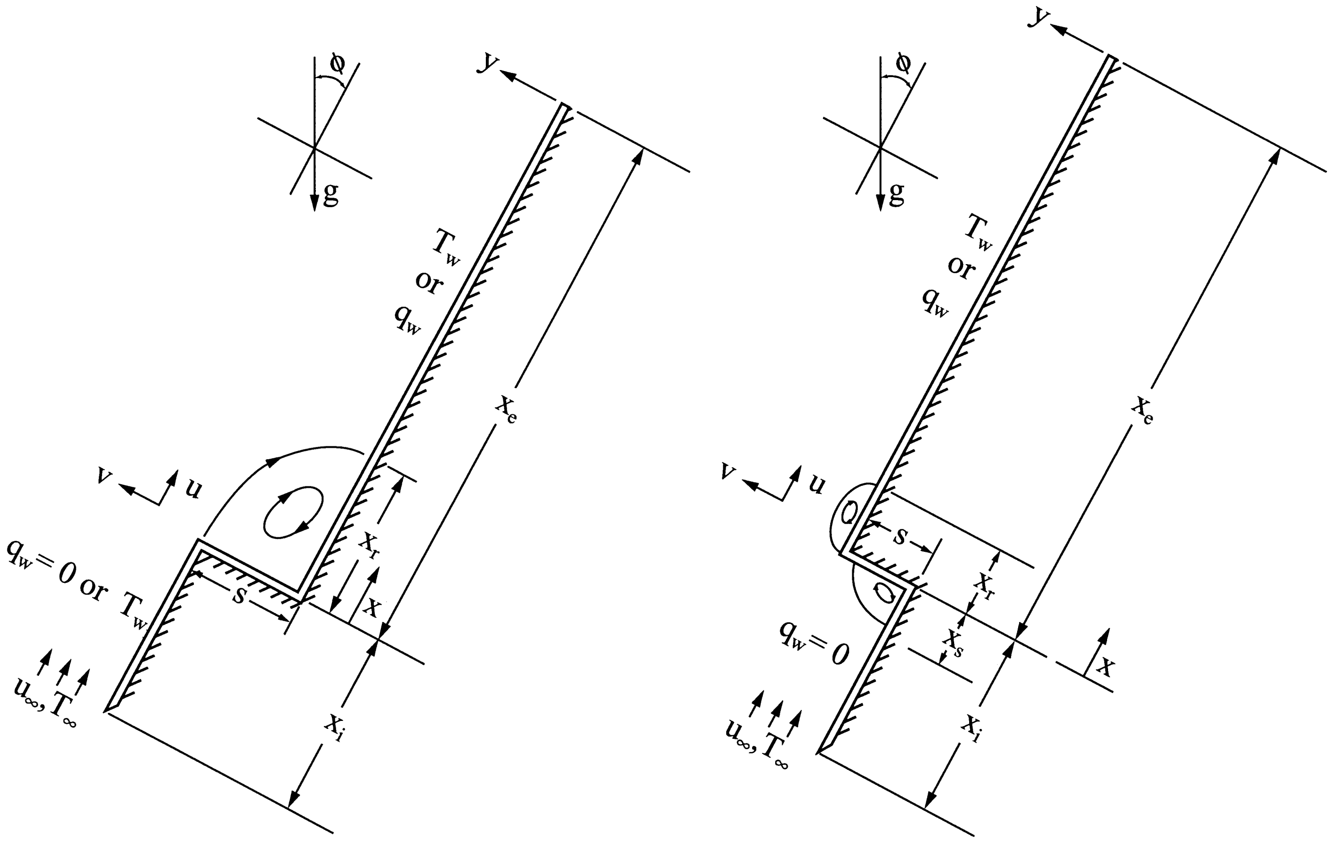
$$\begin{aligned} &U\partial V/\partial X + V\partial V/\partial Y \\ &\quad = -\partial P/\partial Y + (1/Re_\mathrm{s})\bigl(\partial^2 V/\partial X^2 + \partial^2 V/\partial Y^2\bigr) \\ &\qquad + \bigl(Gr_y/Re_\mathrm{s}^2\bigr)\theta \end{aligned} \tag{3}$$

$$\begin{aligned} &U\partial\theta/\partial X + V\partial\theta/\partial Y \\ &\quad = (1/Pr\,Re_\mathrm{s})\bigl(\partial^2\theta/\partial X^2 + \partial^2\theta/\partial Y^2\bigr) \end{aligned} \tag{4}$$

where

$$Gr_x = Gr_\mathrm{s}\cos(\phi), \qquad Gr_y = Gr_\mathrm{s}\sin(\phi)$$

This set of coupled partial differential equations along with the appropriate boundary conditions was solved by using a finite difference, finite volume, finite element, or lattice Boltzmann method.



(a) (b)

Fig. 1. Schematic of the step geometries: (a) backward-facing step, (b) forward-facing step.

2.1. *Backward-facing step geometry*

Laminar mixed convection flow over a backward-facing step has been examined extensively in the past, especially in the last decade, both experimentally and numerically. Both buoyancy-assisting and buoyancy-opposing flows were examined. Vertical, inclined, and horizontal cases were investigated. Both uniform wall temperature and uniform wall heat flux heating conditions were considered. The effect of Prandtl number on the laminar mixed convection flow over a backward-facing step was also examined.

2.1.1. *Buoyancy assisting*

Laminar mixed convection in buoyancy assisting, vertical backward-facing step flows was investigated numerically by Lin et al. [3], Hong et al. [4], Noble [5], and Iwai [6] and experimentally by Baek et al. [7] and Abu-Mulaweh et al. [8,9]. In the numerical studies of Lin et al. [3], Hong et al. [4], and Iwai [6] constant property assumption and Boussinesq approximation were employed, and the solution of the non-dimensional governing conservation set of coupled partial differential equations was obtained by using a finite difference scheme. They have utilized the SIMPLE algorithm, as described by Patankar [10]. On the other hand, Noble [5] employed a relatively new numerical technique called the lattice Boltzmann (LB) method. The lattice Boltzmann method is a kinetic theory-based method for solving transport problems.

Lin et al. [3] studied the buoyancy force effects when the wall behind the vertical backward-facing step was heated and maintained at a constant temperature with an expansion ratio of 2 and a step height of 4.8 mm. The straight wall of the duct was maintained at the inlet air temperature, while the stepped wall of the duct was maintained adiabatic for the upstream wall and for the step itself. The downstream wall behind the step is maintained at a higher, but uniform, temperature. In their study, the full range of laminar mixed convective flow, from pure forced convective flow to the inlet starved convective flow, was examined. They have reported that the influence of the buoyancy force on the velocity distribution is more pronounced than its influence on the temperature distribution. They also found that the buoyancy force significantly affects the recirculation region behind the step and the local Nusselt number (i.e., the local heat transfer rate) downstream of the step. As the value of the buoyancy force parameter increases, as a result of increasing the wall temperature, the local Nusselt number increases and the size of the recirculation region (i.e., the reattachment length) decreases. This is because a higher streamwise buoyancy force induces a higher positive velocity component near the wall. This will decrease, or cancel, the negative velocity component in the recirculation region, causing the reattachment length to decrease. In addition, a secondary recirculation region develops at the corner of the step and grows as the buoyancy force increases (i.e., as the wall temperature increases) until it vents itself to the main flow by pushing the main recirculation region away, thus leaving the heated wall without any reattached flow region (i.e., a detachment of the recirculation region occurs).

Hong et al. [4] investigated numerically laminar mixed convection flow over a vertical backward-facing step in a two-dimensional duct with a heated downstream section of the stepped wall subjected to uniform heat flux (UHF). The upstream wall section of the stepped wall and the backward-facing step were treated as adiabatic surfaces. The straight wall of the duct was maintained at a uniform temperature that is equal to the inlet fluid temperature. In their study, they have examined the effects of Grashof number Gr_s, expansion ratio R, and Reynolds number, Re_s on the Nusselt number, the wall friction coefficient, and the reattachment length. The range of parameters were $0 \leqslant Gr_\mathrm{s}^* \leqslant 2400$, $1.25 \leqslant R \leqslant 4$, and $0 \leqslant Re_\mathrm{s} \leqslant 150$. They found that increasing Grashof number tends to increase the wall friction coefficient and the local Nusselt number on the downstream heated wall, but it decreases the reattachment length. Increasing the expansion ratio will increase the reattachment length when the expansion ratio is less than 2.25, but that trend is reversed when the expansion ratio is larger than 2.25. Also, increasing the expansion ratio increases the fully developed value of the wall friction coefficient and the distance for the flow to reach the fully developed regime, but it decreases the peak and the fully developed Nusselt numbers. Increasing the Reynolds number tends to increase the local Nusselt number and the reattachment length, but it decreases the wall friction coefficient at the heated downstream wall. The effects of Grashof number, expansion ratio, and Reynolds number on the local Nusselt number reported by Hong et al. [4] are presented in Fig. 2. They also developed and reported correlations for the reattachment length as a function of Reynolds number, expansion ratio, and buoyancy force parameter ($\xi = Gr_\mathrm{s}^*/Re_\mathrm{s}^2$, where $Gr_\mathrm{s}^* = g\beta q_\mathrm{w} s^4/k\nu^2$). These correlations can be found in Table 1. In addition, they reported two correlation equations for the peak Nusselt number $Nu_\mathrm{s,max}$ as a function of the Garshof number and the expansion ratio. These correlations are given by:

$$Nu_\mathrm{s,max} = 1.658 + 0.0001483 Gr_\mathrm{s}^* \tag{5}$$

$$Nu_\mathrm{s,max} = 4.862 - 2.172R + 0.3292R^2 \tag{6}$$

The problem of two-dimensional, laminar flow over a backward-facing step in a vertical channel with mixed convection was also solved numerically by Noble [5] utilizing a coupled lattice Boltzmann (LB) model that he has developed for solving buoyancy problems. Noble [5] compared his results of the flow and thermal fields with published results from conventional methods (i.e., finite difference, finite element and finite volume). He reported that the comparisons with those methods validated LB as a quantitative method for thermal problems including mixed convection and that LB method provides alternate techniques for coupled heat transfer and viscous fluid dynamics. However, he found that wiggles in the solution are formed near the corners of the computational domain, and the maximum velocity in the channel is over predicted for the case of mixed convection due to an error in the inlet boundary condition.

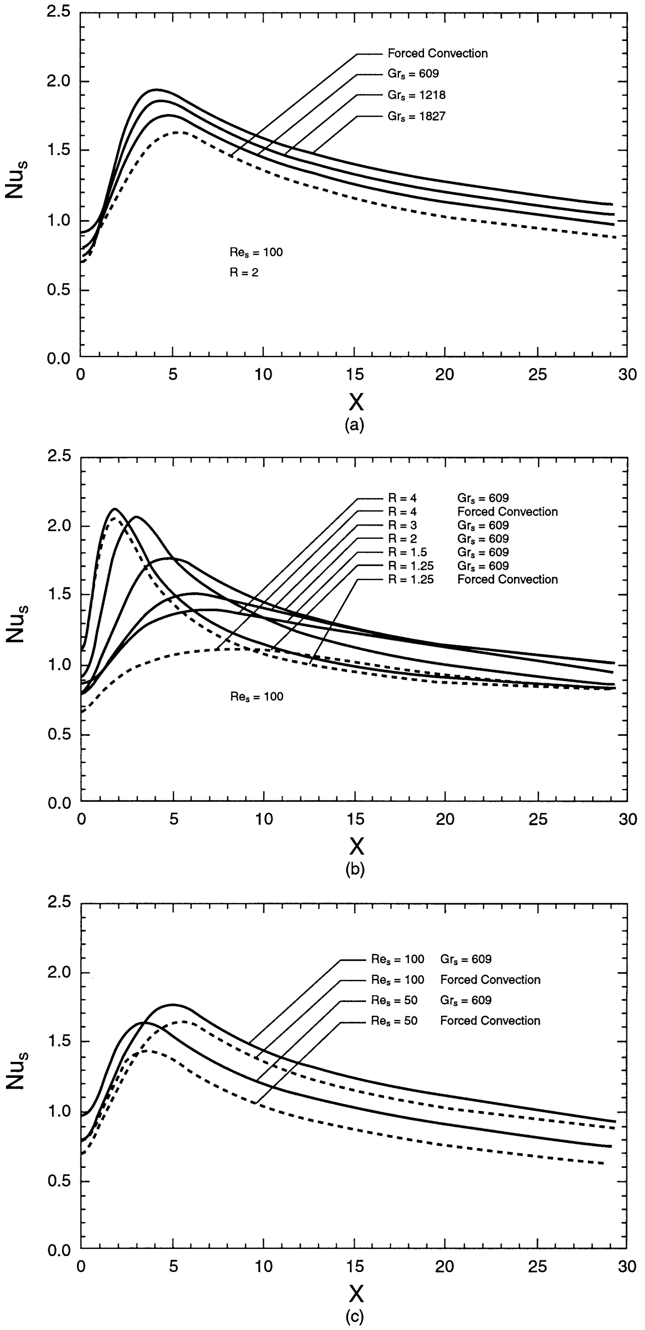


Fig. 2. Effects of (a) Grashof number, (b) expansion ratio, (c) Reynolds number on the axial variation of the local Nusselt number, Ref. [4].

Table 1
Rettachment length correlations for buoyancy-assisting flow

Author	Step	Heating condition	Ranges	Correlation
Hong et al. [4]	Vertical backward	UHF	$20 \leqslant Re_\mathrm{s} \leqslant 150$ $1.25 \leqslant R \leqslant 4$ $0 \leqslant \xi \leqslant 0.24$	$X_\mathrm{r} = X_{\mathrm{r,forced}}\{1 + \{7.188 - 4.047R - 1.758Re_\mathrm{s})\xi\}$ $X_{\mathrm{r,forced}} = a_0(R) + a_1(R)Re_\mathrm{s}$ $a_0(R) = -1.252 + 1.345R - 0.1691R^2$ $a_1(R) = -0.02751 + 0.04572R - 0.006414R^2$
Baek et al. [7]	Vertical backward	UWT	$0.38\ \mathrm{cm} \leqslant s \leqslant 1.0\ \mathrm{cm}$ $0.37\ \mathrm{m{\cdot}s^{-1}} \leqslant u_\mathrm{o} \leqslant 0.72\ \mathrm{m{\cdot}s^{-1}}$ $0\,^\circ\mathrm{C} \leqslant \Delta T \leqslant 3\,0\,^\circ\mathrm{C}$	$X_\mathrm{r} = (2.24 + 0.022Re_\mathrm{s})\exp[-42\xi(s/x_\mathrm{i})^{-0.186}]$
Abu-Mulaweh et al. [8]	Vertical backward	UHF	$0.35\ \mathrm{cm} \leqslant s \leqslant 0.8\ \mathrm{cm}$ $50 \leqslant Re_\mathrm{s} \leqslant 330$ $0 \leqslant \xi \leqslant 1.4 \times 10^{-2}$	$X_\mathrm{r} = X_{\mathrm{r,forced}}\exp(-7.5\xi^{1/2})$ $X_{\mathrm{r,forced}} = 0.023(\delta_\mathrm{s}/s)Re_\mathrm{s} + f(s/x_\mathrm{i})$ $f(s/x_\mathrm{i}) = 8995.8(s/x_\mathrm{i})^2 - 114.2(s/x_\mathrm{i}) - 3.4$
Hong et al. [14]	Inclined backward	UHF	$Re_\mathrm{s} = 100,\ R = 2,$ $Gr_\mathrm{s}^* = 609,\ Pr = 0.712$ $0 \leqslant \phi \leqslant 360$ deg	$X_\mathrm{r} = \left[5.462 - 1.62\cos(\phi)\right]$ $\times\left[0.95 + 0.05\cos(2\phi)\right]\left[1 + 0.02\sin(\phi)\right]$
Hong et al. [14]	Vertical backward	UHF	$Re_\mathrm{s} = 100,\ R = 2,$ $Gr_\mathrm{s}^* = 609$ $0.07 \leqslant Pr \leqslant 100$	$X_\mathrm{r} = 4.9 - 1.7\exp(-Pr/0.4)$
Soong and Hsueh [18]	Horizontal backward	UWT w/injection	$R = 1.5,\ Re_\mathrm{s} = 50,\ l_j = 0.2$ $0 \leqslant \xi \leqslant 0.4$ $0 \leqslant v_j \leqslant 0.2$	$X_\mathrm{r} = 4.2525 + 0.4158v_j + 2.5017\xi$
			$R = 1.5,\ Re_\mathrm{s} = 100,\ l_j = 0.2$ $0 \leqslant \xi \leqslant 0.4$ $0 \leqslant v_j \leqslant 0.2$	$X_\mathrm{r} = 6.6446 + 3.5701v_j + 6.7160\xi$
Abu-Mulaweh et al. [31]	Horizontal forward	UWT	$1.27\ \mathrm{cm} \leqslant s \leqslant 1.75\ \mathrm{cm}$ $1.10\ \mathrm{cm} \leqslant \delta_\mathrm{s} \leqslant 1.78\ \mathrm{cm}$ $270 \leqslant Re_\mathrm{s} \leqslant 570$	$X_\mathrm{r} = f_1(s/x_\mathrm{i})(\delta_\mathrm{s}/s)Re_\mathrm{s} + g_1(s/x_\mathrm{i})$ $f_1(s/x_\mathrm{i}) = 0.3735(s/x_\mathrm{i}) + 0.00265$ $g_1(s/x_\mathrm{i}) = -23.989(s/x_\mathrm{i}) - 6.077$ $X_\mathrm{s} = f_2(s/x_\mathrm{i})(\delta_\mathrm{s}/s) + g_2(s/x_\mathrm{i})$ $f_2(s/x_\mathrm{i}) = 1219.7(s/x_\mathrm{i})^2 - 201.2(s/x_\mathrm{i}) + 5.52$ $g_2(s/x_\mathrm{i}) = -3.057(s/x_\mathrm{i}) + 4.739$
Abu-Mulaweh et al. [32]	Vertical forward	UWT	$1.27\ \mathrm{cm} \leqslant s \leqslant 1.75\ \mathrm{cm}$ $1.10\ \mathrm{cm} \leqslant \delta_\mathrm{s} \leqslant 1.78\ \mathrm{cm}$ $270 \leqslant Re_\mathrm{s} \leqslant 570$ $0 \leqslant \xi \leqslant 6.8 \times 10^{-2}$	$X_\mathrm{r} = X_{\mathrm{r,forced}}\exp(-4.1\xi^{1/2})$ $X_{\mathrm{r,forced}} = f_1(s/x_\mathrm{i})(\delta_\mathrm{s}/s)Re_\mathrm{s} + g_1(s/x_\mathrm{i})$ $f_1(s/x_\mathrm{i}) = 0.3735(s/x_\mathrm{i}) + 0.00265$ $g_1(s/x_\mathrm{i}) = -23.989(s/x_\mathrm{i}) - 6.077$ $X_\mathrm{s} = f_2(s/x_\mathrm{i})(\delta_\mathrm{s}/s) + g_2(s/x_\mathrm{i})$ $f_2(s/x_\mathrm{i}) = 1219.7(s/x_\mathrm{i})^2 - 201.2(s/x_\mathrm{i}) + 5.52$ $g_2(s/x_\mathrm{i}) = -3.057(s/x_\mathrm{i}) + 4.739$

A three-dimensional numerical study was carried out by Iwai et al. [6] for laminar mixed convection flow over a vertical backward-facing step in a duct with a heated downstream wall subjected to uniform heat flux. The step and the rest of the walls were assumed to be adiabatic. In their study, they have examined the effect of buoyancy (assisting) on the flow and thermal fields. The numerical calculations were carried out for a Reynolds number of 125, expansion ratio of 2, aspect ratio of 16, buoyancy parameter ranging from zero to 0.12. They have found that maximum Nusselt numbers are located symmetrically near both side walls and not on the center line of the heated wall, similar to what they have found in their studies of the pure forced convection case (Iwai et al. [11,12]). They reported that two-dimensional computations are effective in predicting this type of phenomena. However, when three-dimensionality of the flow and thermal fields predominates ($\xi > 0.06$), a three-dimensional computation becomes necessary. They also found, similar to the two-dimensional studies results, that the reattachment point and the peak Nusselt number move closer to the step as the buoyancy-assisting level increases, whereas the secondary recirculation region, which develops at the corner of the step, becomes larger. In addition, the peak Nusselt number location always appears downstream of the reattachment point and the distance between them increases with increasing buoyancy-assisting level.

Measurements and predictions of buoyancy-assisting laminar mixed convection boundary layer flow adjacent to a two-dimensional, heated vertical backward-facing step for both uniform wall temperature and uniform wall heat flux heating conditions were reported, respectively, by Baek et al. [7] and Abu-Mulaweh et al. [8]. In both studies the upstream wall of the step and the step itself were kept adiabatic, and the wall downstream of the step was heated. Measurements of the fluid velocity and temperature distributions were obtained by utilizing a laser-Doppler velocimeter and a

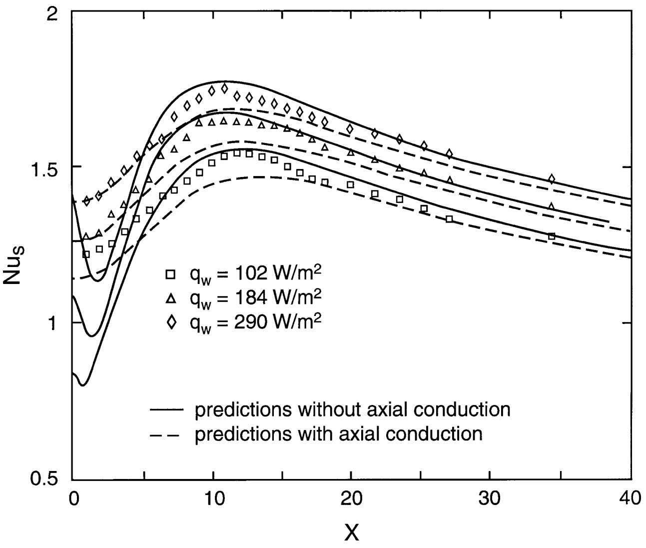


Fig. 3. Effects of buoyancy force on the axial variation of the local Nusselt number, Ref. [8].

cold-wire anemometer, respectively. Both studies found that as the buoyancy-assisting force increases, the reattachment length X_r decreases but the local Nusselt number (i.e., local heat transfer rate) increases and the location of its maximum value moves closer to the step. In addition, Abu-Mulaweh et al. [8] found that the local Nusselt number to be lower in the region where $X < X_r$ and higher in the region where $X > X_r$ for higher free stream velocity. Also, Abu-Mulaweh et al. [8] found that because of the existence of a large axial wall temperature gradient, the axial conduction along the constant heat flux surface is very significant in the recirculation region and cannot be neglected. In order to accurately predict the measured data numerically, a coupling of the convective heat transfer from the wall to the fluid with the axial conduction along the constant heat flux wall was found to be necessary. Fig. 3 presents the results reported by Abu-Mulaweh et al. [8] regarding the effects of buoyancy force on the axial variation of the local Nusselt number. Both Baek et al. [7] and Abu-Mulaweh et al. [8] developed and reported correlation equations for the reattachment length. These correlation equations can be found in Table 1.

The effects of heating the upstream wall and the step of two-dimensional, vertical backward-facing step geometry on the flow and heat transfer characteristics in the region downstream of the step were reported by Abu-Mulaweh et al. [9]. Four different upstream wall/step heating conditions were examined. The first case TBC-1 corresponds to $T_{w1} - T_\infty = T_w - T_\infty$, the second case TBC-2 corresponds to $T_{w1} - T_\infty = 2(T_w - T_\infty)$, the third case TBC-1/2 corresponds to $T_{w1} - T_\infty = (T_w - T_\infty)/2$, and the fourth case TBC-A corresponds to adiabatic upstream wall/step (where T_w is the downstream wall temperature, T_{w1} is the upstream wall/step temperature, and T_∞ is the freestream temperature). It was found that the thermal boundary conditions of the upstream wall and the step affect significantly the temperature and velocity distributions, the local heat transfer rate, and the reattachment length for the mixed convection regime downstream of the step. It was found that the recirculation region remains attached to the heated wall downstream of the step when the upstream wall and the step are heated to the same or different uniform temperature as the downstream wall. It may however, detach from the heated downstream wall at high buoyancy levels when the upstream wall and the step are treated as adiabatic surfaces. The reattachment length was found to decrease as the buoyancy level is increased, with those for the higher upstream heating levels having the smaller reattachment lengths (see Fig. 4). When the upstream wall and the step are heated to a higher temperature than the downstream wall, the velocity distribution downstream of the step may exhibit an overshoot over its free stream value. This behavior is shown in Fig. 5. Also, it was found that the heat transfer from the down stream wall to the fluid decreases as the upstream wall and the step are heated. The effect of upstream wall/step thermal condition on the local rate of heat transfer downstream of the step is shown in Fig. 6.

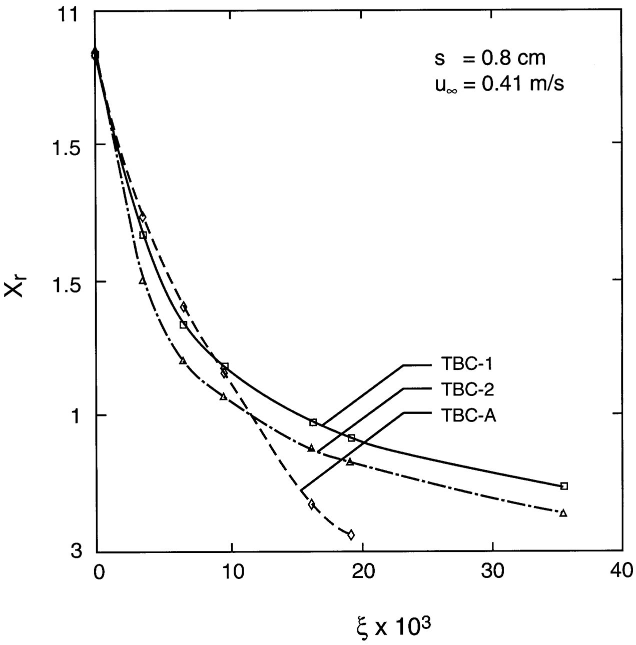


Fig. 4. Effects of upstream wall heating on the reattachment length, Ref. [9].

The effect of inclination angle on laminar mixed convection flow over a backward-facing step was investigated numerically by Lin et al. [13], Hong et al. [14], and Iwai et al. [15], and experimentally by Abu-Mulaweh et al. [16]. These studies found that the inclination angle affects significantly the flow and thermal fields of laminar mixed convection downstream of a backward-facing step. They found that an increase in the inclination angle from the vertical will result in a decrease in the local Nusselt number (i.e., a decrease in the local heat transfer rate) and an increase in the reattachment length and the location of the maximum Nusselt number. This increase is due to a decrease in the streamwise buoyancy force caused by the increase in the inclination an-

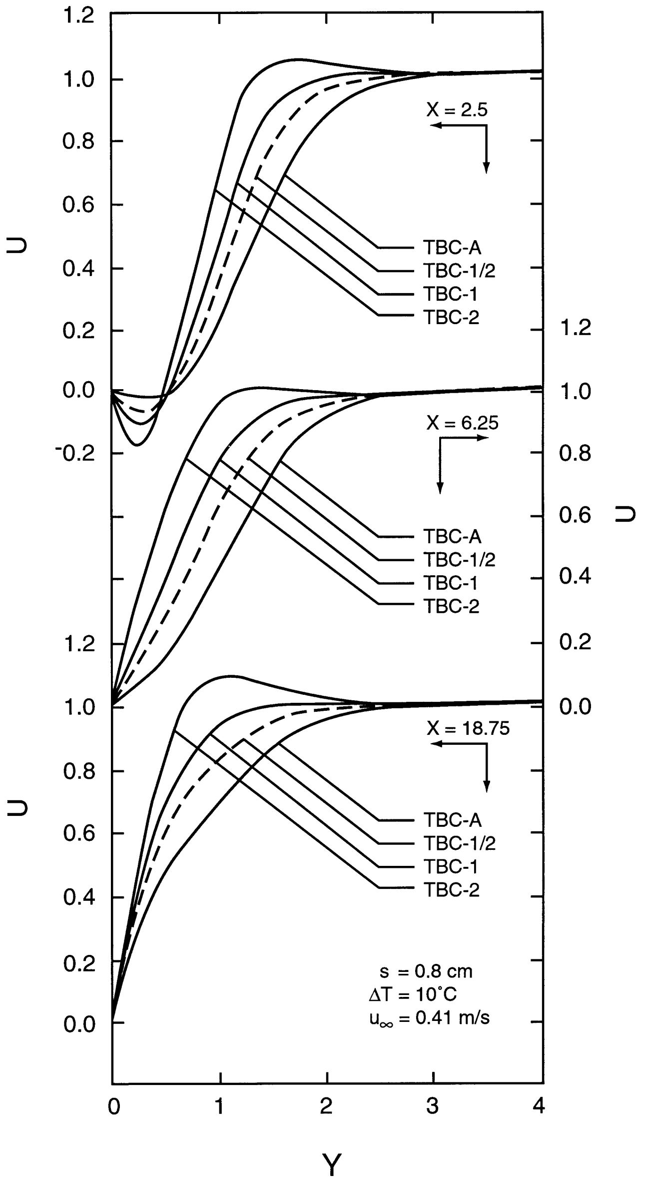


Fig. 5. Effects of upstream wall heating on the velocity distributions, Ref. [9].

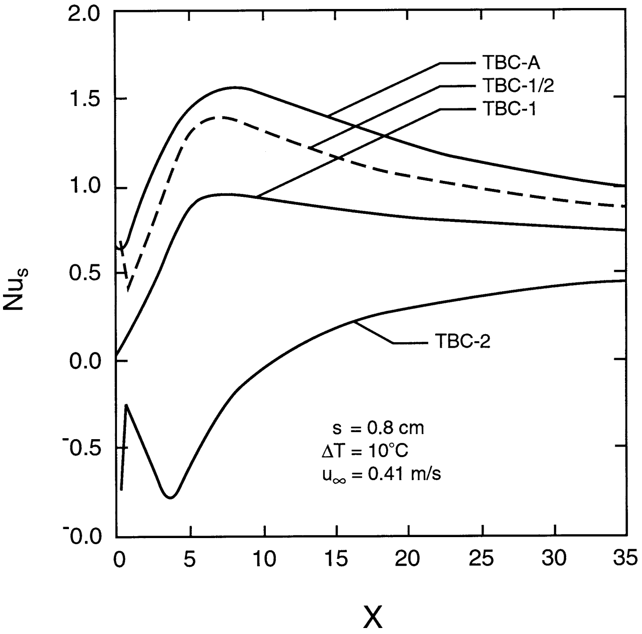


Fig. 6. Effects of upstream wall heating on the local Nusselt number, Ref. [9].

gle. Correlation equations for the reattchment length X_r, the location of the peak Nusselt number X_n, and the maximum Nusselt number $Nu_{s,\max}$ as a function of the inclination angle were developed by Hong et al. [14]. The correlation for the reattachment length can be found in Table 1. The correlations for the location of the peak Nusselt number and the maximum Nusselt number are given, respectively, by:

$$X_n = \left[5.472 - 0.6535\cos(\phi)\right]\left[0.98 + 0.02\cos(2\phi)\right] \times \left[1.0 + 0.02\sin(\phi)\right] \tag{7}$$

$$Nu_{s,\max} = \left[1.01 - 0.01\cos(2\phi)\right]\left[1.752 + 0.151\cos(\phi)\right] \tag{8}$$

It should be noted that both Hong et al. [14] and Iwai et al. [15] considered the heated downstream wall to be maintained at a uniform heat flux. However, Iwai et al. [15] carried out three-dimensional numerical simulations in their study of the effects of the inclination angle on laminar mixed convective flows over a backward-facing step. Iwai et al. [15] examined the effect of two inclination angles: the pitch angle of the duct and the rolling angle of the duct. The numerical computations were carried out for a Reynolds number of $Re_s = 125$, expansion ratio of $R = 2$, aspect ratio of $AR = 16$, buoyancy parameter of $\xi = 0.03$. They reported that both the pitch and rolling angles significantly affect the flow and thermal fields. The effect of the pitch angle of the duct was similar to the effect that was reported by Lin et al. [13] and Hong et al. [14] (two-dimensional computations). However, in the three-dimensional computations, when the pitch angle is changed and keeping the rolling angle to be zero degree, positions of the maximum Nusselt numbers were found symmetrically at the positions on the heated wall near the two side walls, similar to the cases of pure forced convection. They have found that when the rolling angle of the duct was varied and keeping the pitch angle to be 90 deg, the flow and thermal fields became asymmetric about the duct centerline. The downwash flow directed toward the heated wall is prominent only near the lower side wall, resulting only in one prominent peak of the Nusselt number there. They also found that the maximum Nusselt number appears at the most upstream position in the case of the rolling angle of 90 deg and takes the greatest value.

For the case of mixed convection flow over horizontal backward-facing step case (i.e., $\phi = 90$ deg), Abu-Mulaweh et al. [16] found that the buoyancy force resulting from the heating of the downstream wall has a negligible effect on the velocity and temperature distributions, the reattachment length, and the rate of heat transfer, but it influences the onset/start of vortex instability. The onset of vortex instability

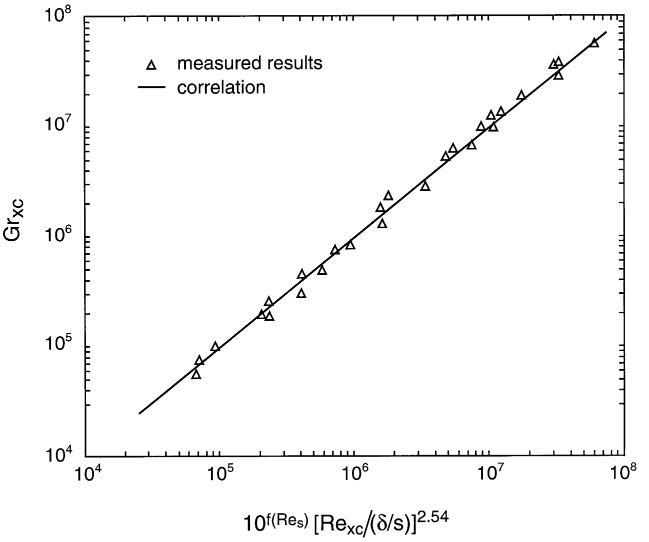


Fig. 7. Critical Grashof and Reynolds numbers for the onset of vortex instability.

limits the applicability of the two-dimensional laminar flow solution for predicting the flow and heat transfer characteristics in this geometry. The critical length downstream of the step at which vortex instability starts to occur is affected by the step height, the free stream velocity, and the temperature difference between the heated wall and the free stream. Cheng and Kimura [17] reported in their qualitative study, using flow visualization, of mixed convective instability for laminar boundary-layer flow over a horizontal backward-facing step that the longitudinal vortex rolls appear immediately behind the backward-facing step. This is due to the large step height that was used in their study ($s = 1.3$ cm), for which the flow appears to be unstable at any given heating. Abu-Mulaweh et al. [16] determined that for horizontal backward-facing step geometry with isothermal flow (i.e., no wall heating), the flow becomes unstable and the onset of instability occurs right behind the step when the step is larger than $s = 1.0$ cm for a free stream velocity higher than $0.285\ \mathrm{m\cdot s^{-1}}$. The axial locations where the onset of vortex instability occurs under different flow and heating conditions were correlated by the following equation as shown in Fig. 7:

$$Gr_{xc}/Re_{xc}^{2.5} = (s/\delta_s)^{2.25}\exp\left[f(Re_s)\right] \tag{9}$$

$$f(Re_s) = 2.206\times 10^{-4}Re_s^2 - 0.0896Re_s + 6.01 \tag{10}$$

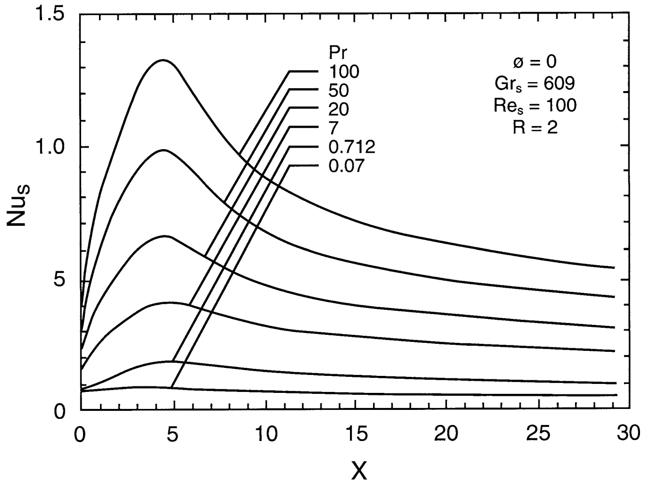


Fig. 8. Effects of Prandtl number on the axial variation of the local Nusselt number, Ref. [14].

The only study that has dealt with the effect of the Prandtl number on the flow and heat transfer characteristics of laminar mixed convection over a backward-facing step is that of Hong et al. [14]. In their study, they have examined the influence of Prandtl number on the flow and thermal field characteristics of buoyancy-assisting flow over a vertical backward-facing step. They have found that increasing the Prandtl number will increase the Nusselt number and the reattachment length, but it will decrease the wall friction coefficient. Fig. 8 shows the effect of Prandtl number on the local Nusselt number (i.e., the local heat transfer rate) downstream of the step. Correlation equations for the Reattachment length X_r, the location of the peak Nusselt number X_n, and the maximum Nusselt number $Nu_{s,\max}$ as a function of Prandtl number were developed by Hong et al. [14]. The correlation equation for the reattachment length can be found in Table 1. The correlation equations for the location of the peak Nusselt number and the maximum Nusselt number are given, respectively, by:

$$X_n = 3.53 + 1.71Pr \qquad 0.07 \leqslant Pr \leqslant 0.712 \tag{11a}$$

$$X_n = 4.753 - 0.0091Pr + 6.786\times 10^{-5}Pr^2 \qquad 0.712 \leqslant Pr \leqslant 100 \tag{11b}$$

$$Nu_{s,\max} = 0.52 + 1.29\sqrt{Pr} \tag{12}$$

The effects of cold fluid injection on mixed convection flow over a horizontal backward-facing step were investigated numerically by Soong and Hsueh [18]. The numerical computations were carried out for a channel expansion ratio of $R = 1.5$ and main flow Reynolds numbers of $Re_s = 50$ and 100. They have found that both the effects of fluid injection and buoyancy increase the size of the recirculation region behind the step. They developed and reported correlation equations for the reattachment length for various conditions of injection velocity and buoyancy. These correlation equations can be found in Table 1. They also found that the cold fluid injection enhances the heat transfer on the backward-facing wall and slightly reduces that on the bottom wall. The fluid injection moves the reattachment point downstream and therefore the location where the peak of the Nusselt number is located.

At the 1993 ASME Winter Annual Meeting, the Aerospace Heat Transfer Committee (K-12) of the Heat Transfer Division of the ASME held a technical session for a benchmark heat transfer problem (see Blackwell and Armaly [1, 2]) to test the accuracy and the claims of the many computational fluid dynamics (CFD) codes. The benchmark prob-

Table 2
Reattachment length on heated wall, X_r (Benchmark problem)

Author	Type	$Gr_\mathrm{s} = 0$	$Gr_\mathrm{s} = 1000$
Acharya et al.	FD	4.975	2.972
Chopin	FE	4.61	2.99
Choudhury and Woolfe	FV	5.02	3.21
Cochran et al.	FE	4.991	2.997
Cochran et al.	FV	5.325	3.237
Dyne et al.	FE	4.894	2.976
Hong et al.	FD	4.94	2.98
Kasz et al.	FD	4.29	2.34
McHugh et al.	FD	4.976	2.91
Sanchez and Vradis	FD	4.953	2.945
Torczynski	FE	4.984	2.977
Torczynski	SE	4.985	2.979

lem was to solve a steady-state two-dimensional, buoyancy-assisted, mixed convection flow of a laminar Newtonian fluid in a vertical channel with a backward-facing step. The upstream wall and the step itself were treated as adiabatic surfaces, while the down stream wall of the step was considered to be heated at uniform and constant temperature. The straight wall of the channel was considered to be maintained at the inlet fluid temperature. Eleven papers [19–29] were contributed in which the benchmark problem was solved numerically by utilizing different codes such as FLOTRAN, FLUENT, TEACH, COUGAR, KAMELEON-II, FIDAP, NEKTON, CUTEFLOWS, and QMRCGS. The authors solved the benchmark problem and submitted results for the following parameters: $Re_\mathrm{s} = 100$, $Pe_\mathrm{s} = 70$, and $Gr_\mathrm{s} = 1000$. A comparison of the reattachment length results of the benchmark papers are presented in Table 2. Their predictions of the reattachment length for laminar, buoyancy-assisted mixed convection flow of the specified benchmark problem were very close to each other ($X_\mathrm{r} = 2.91 \sim 3.237$) with the exception of Kasz et al. [26] where they predicted the reattachment length, $X_\mathrm{r} = 2.34$.

2.1.2. Buoyancy-opposing

In this case, the buoyancy-induced flow adjacent to the heated wall is in a direction opposite to the main forced flow. To the best knowledge of the author, the study of Abu-Mulaweh et al. [30] is the only study, which has been reported in the open literature, of laminar mixed convection in the backward-facing step that has dealt with the buoyancy-opposing case. The backward-facing step geometry used in the experimental study of Abu-Mulaweh et al. [30] was identical to that used by Baek et al. [7], except that the orientation of the air tunnel was changed by 180 deg to create buoyancy-opposing flow conditions. Abu-Mulaweh et al. [30] also modeled this experimental geometry for numerical simulation by using a finite difference scheme, embodied in the computer code TEACH using the SIMPLE algorithm, as described by Patankar [10].

Abu-Mulaweh et al. [30] found that the main flow and the buoyancy-induced flow interact with each other, and for low buoyancy levels, $\xi < 4.4 \times 10^{-3}$, the flow remains laminar throughout the test section (see Fig. 9). In this laminar regime, the length of the recirculation region downstream of the backward-facing step increases rapidly as the buoyancy level increases due to the buoyancy-induced velocity, which is opposing the forced flow direction adjacent to the heated wall. However, they found that the interaction between the main forced flow and the buoyancy-induced flow causes the flow to become turbulent downstream of the recirculation region as the buoyancy level increases. This caused the numerical scheme they have employed, which did not include turbulence modeling, to fail in predicting their measured results in the laminar recirculation region. The development of turbulent flow downstream of the recirculation region made the numerical model unstable for predicting this flow. It should be noted that in this flow regime, the flow inside the recirculation region remains laminar, but the length of the recirculation region decreases rapidly with increasing buoyancy level as a result of the transition from laminar to turbulent flow (see Fig. 9). They also found that, in contrast to the buoyancy-assisting flow case, the local Nusselt number (i.e., local heat transfer rate) downstream of the step decreases as the buoyancy-opposing force increases (i.e., with increasing wall heating). The effect of buoyancy on the local Nusselt number downstream of the step is shown in Fig. 10.

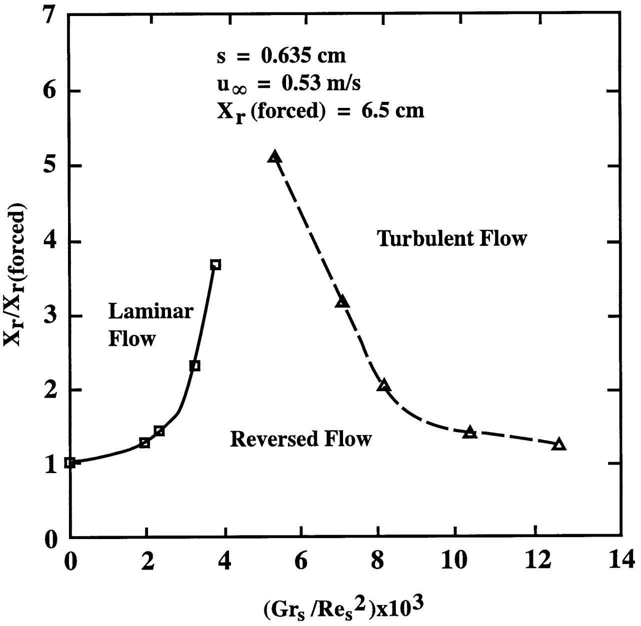


Fig. 9. Effects of buoyancy-opposing force on the reattachment length, Ref. [30].

As it was discussed earlier in this paper, Hong et al. [14] and Iwai et al. [15] numerically examined the effect of inclination angle on the laminar mixed convection flow over a backward-facing step. In both studies they covered the buoyancy-opposing case ($\phi = 180$ deg). The author believes that the results reported by Hong et al. [14] for the case of $\phi = 180$ deg are doubtful. Because for the conditions reported in their two-dimensional numerical simulation,

Table 3
Laminar mixed convection flow over backward-facing step

	As the parameter increases						
	Step height	Velocity	Buoyancy-assisting	Buoyancy-opposing	Inclination angle	Prandtl number	Expansion ratio
Local heat transfer rate	Increases	Increases	Increases	Decreases	Decreases	Increases	Decreases
Reattachment length, x_r	Increases	Increases	Decreases	Decreases	Increases	Increases	$R < 2.25$: Increase $R > 2.25$: Decreases

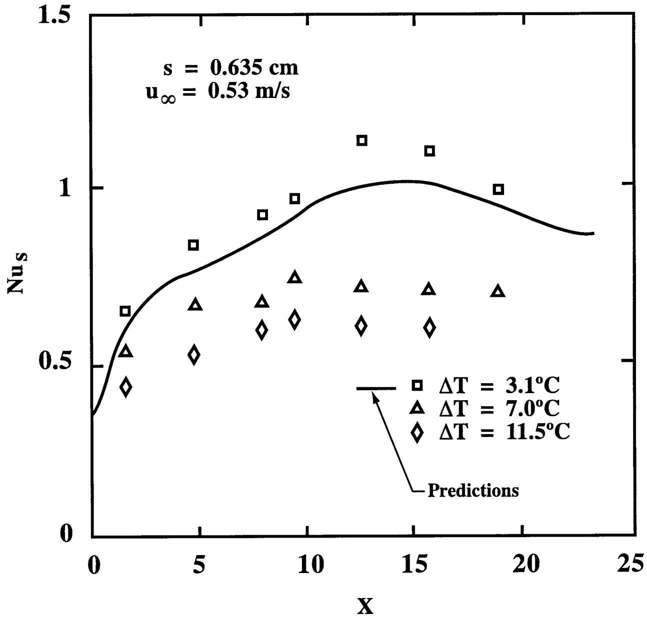


Fig. 10. Effects of wall temperature on the axial variation of the local Nusselt number, Ref. [30].

turbulent flow develops downstream of the recirculation region, and the numerical scheme they employed did not include turbulence modeling. On the other hand, Iwai et al. [15] reported in their three-dimensional simulation that the computation was no longer stable when the buoyancy-opposing level $\xi > 0.05$.

In summary, the effects of step height, velocity, buoyancy due to downstream wall heating (assisting and opposing), inclination angle, Prandtl number, and expansion ratio on the reattachment length and local Nusselt number of laminar mixed convection over a backward-facing step are tabulated in Table 3.

2.2. Forward-facing step geometry

For a flow past a forward-facing step, depending on the magnitude of the flow Reynolds number and the thickness of the momentum boundary layer at the step, one or two separated flow regions may develop adjacent to the step. A recirculation region can develop downstream of the step and another can develop upstream of the step. These separated flow regions make this geometry more complicated to study than the backward-facing step in which only one separated flow region occurs behind the step. Owing to this fact, very limited number of studies has examined the laminar mixed convection flow over a forward-facing step in contrast to the backward-facing step geometry. Both buoyancy-assisting and buoyancy-opposing flows were investigated. Horizontal and vertical cases were examined. Only one heating condition, uniform wall temperature, was considered. None was reported for uniform wall heat flux heating condition. There are no studies in the open literature that can be found which examined the effect of inclination angle or Prandtl number on laminar mixed convection over a forward-facing step.

2.2.1. Buoyancy-assisting

There are only two studies, that can be found in the open literature, that have examined buoyancy-assisting, two-dimensional, laminar mixed convection flow over a forward-facing step. They are the studies of Abu-Mulaweh et al. [31] for the case of horizontal forward-facing step and Abu-Mulaweh et al. [32] for the case of vertical forward-facing step. In both studies, the step and the wall downstream of the step were heated and maintained at a uniform temperature while the upstream wall was adiabatic. Abu-Mulaweh et al. [31,32] also modeled these two experimental geometry for numerical simulation by using a finite difference scheme, embodied in the computer code TEACH using the SIMPLE algorithm, as described by Patankar [10].

In the case of laminar mixed convection flow over a horizontal forward-facing step, Abu-Mulaweh et al. [31] reported that the step height and inlet velocity (Reynolds number) affect significantly the size of the recirculation regions and the local heat transfer rate downstream of the step. On the other hand, the buoyancy force resulting from the heating of the downstream wall has negligible effect on these parameters, because that buoyancy force has no streamwise component. They have found that the size of the recirculation regions increases, and the heat transfer rate from the heated downstream wall decreases as the step height increases, and that both parameters increase as the inlet velocity increases. They also have observed that when $\delta_s/s > 1.15$, the upper recirculation region disappears and only one recirculation region exists upstream of the step, and that when $\delta_s/s < 0.7$ transition from laminar flow to turbulent flow starts to develop in the upper recirculation region downstream of the step. In addition, Abu-Mulaweh et al. [31] developed and reported correlation equations for both the reattachment length of the upper recirculation region (downstream of the step) X_r and the length of the upstream recirculation region (in front of the step) X_s in terms of the calculated thickness of the approaching

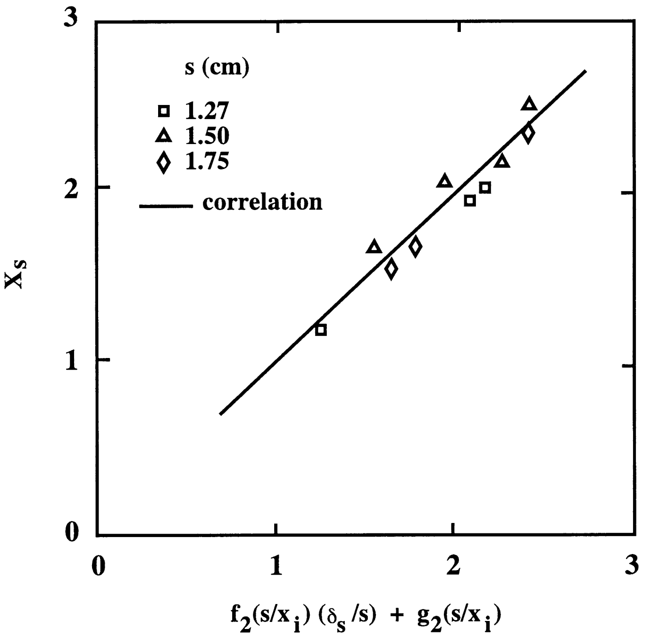


Fig. 11. Correlation for the reattachment length of the upper recirculation region (downstream of the step), Ref. [31].

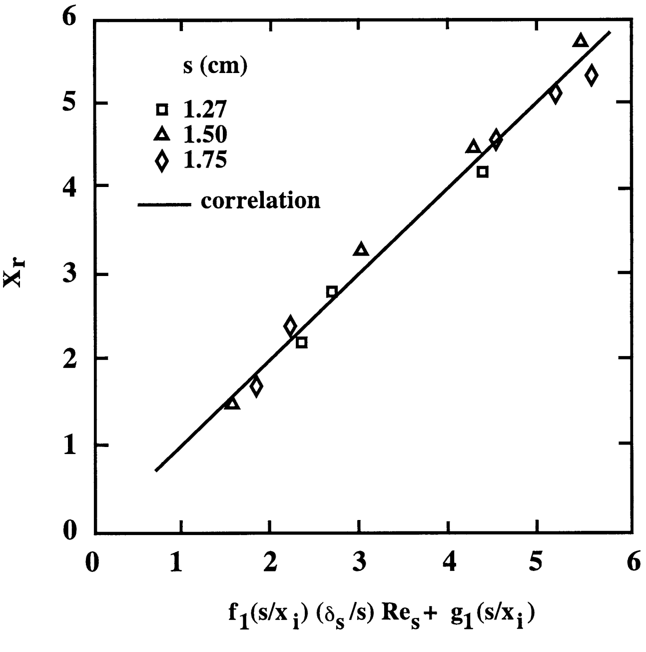


Fig. 12. Correlation for the separation length of the upstream recirculation region (in front of the step), Ref. [31].

momentum boundary layer at the step δ_S, the step height s, and Reynolds number Re_S. These correlation equations can be found in Table 1 and shown graphically in Figs. 11 and 12.

In the case of laminar mixed convection flow over a vertical forward-facing step, Abu-Mulaweh et al. [32] reported that the buoyancy force resulting from the heating of the wall downstream of the step significantly affects the local Nusselt number and the size of the recirculation region downstream of the step. On the other hand, because the upstream wall was kept adiabatic, the recirculation region upstream of the step is not affected by the heating of the downstream wall. They found that as the wall heating increases, the local Nusselt number increases while the size of the recirculation region downstream of the step decreases. Empirical correlation equation to account for the effect of downstream wall heating (i.e., buoyancy-assisting force) on the reattachment length downstream of the step was developed and reported by Abu-Mulaweh et al. [32]. This correlation equation can be found in Table 1 and shown in Fig. 13. As can be seen from the figure, the reattachment length, in the case of mixed convection flow, decreases exponentially with increasing buoyancy-assisting force.

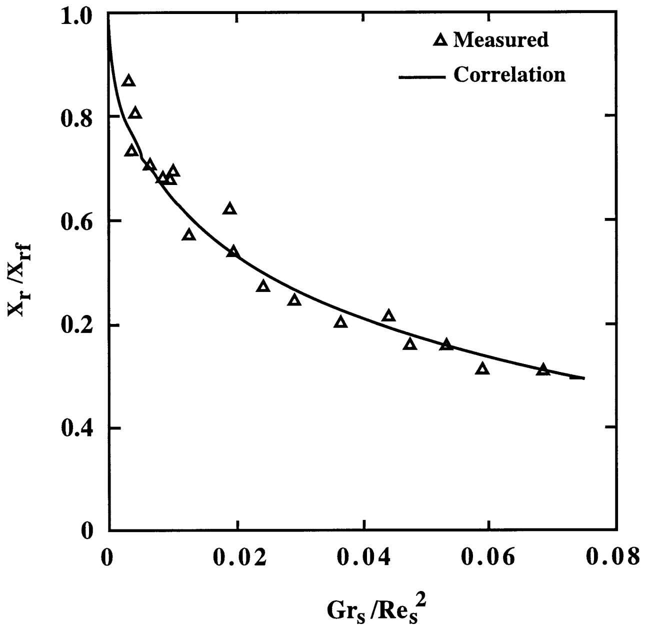


Fig. 13. Reattachment length of the upper recirculation region (downstream of the step) for mixed convection, Ref. [32].

2.2.2. Buoyancy-opposing

The study of Abu-Mulaweh et al. [33] is the only study that can be found in the open literature, which has dealt with buoyancy-opposing, laminar mixed convection flow over a forward-facing step. The forward-facing step geometry used in the experimental study of Abu-Mulaweh et al. [33] was identical to that used by Abu-Mulaweh et al. [32], except that the orientation of the air tunnel was changed by 180 deg to create buoyancy-opposing flow conditions. Abu-Mulaweh et al. [33] also modeled this experimental geometry for numerical simulation by using a finite difference scheme, embodied in the computer code TEACH using the SIMPLE algorithm, as described by Patankar [10].

In the study of Abu-Mulaweh et al. [33], the effects of opposing buoyancy force (i.e., $\Delta T > 0\,^{\circ}\mathrm{C}$) were examined for the case where $\delta_S/s > 1.15$ with laminar approaching flow. They selected this regime because it exhibits a laminar flow region downstream of the step which can be influenced

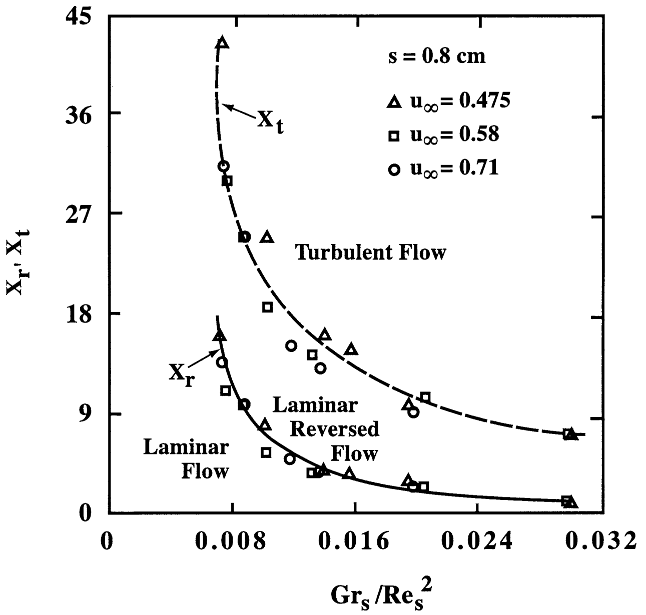


Fig. 14. Effect of buoyancy-opposing force on the reciculation region downstream of the step, Ref. [33].

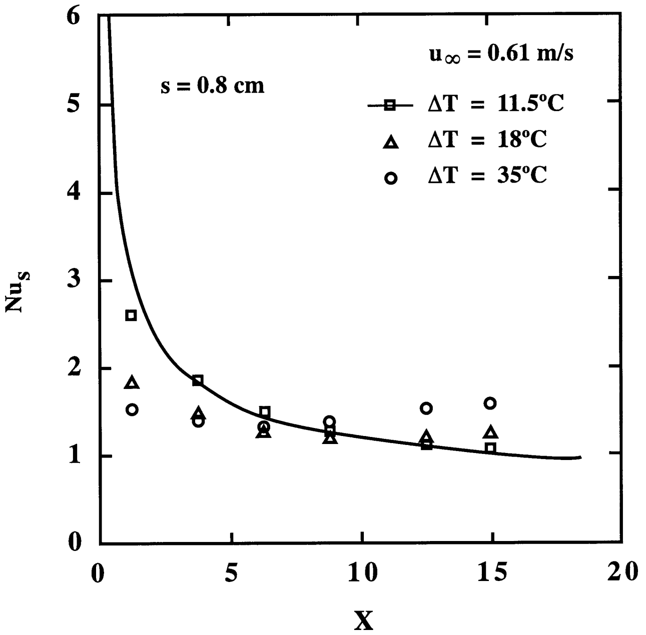


Fig. 15. Effects of buoyancy-opposing force on the axial variation of the Nusselt number, Ref. [33].

significantly by increasing the buoyancy-opposing force. Also, it allows the opportunity to study the transition of flow from laminar to turbulent which is caused by the buoyancy-opposing force. They have found that the interaction of the two flows (i.e., the downward laminar forced flow and the buoyancy-opposing induced flow in the opposite direction) causes the development of three distinct flow regions adjacent to the heated plate downstream of the step: a laminar flow region without any recirculating flow, a laminar recirculating flow region, and a turbulent flow region (see Fig. 14). They have observed that as the buoyancy-opposing force increases as a result of increase in the wall temperature of the downstream wall, the laminar recirculating flow region moves closer to the step, its length decreases, and the turbulent flow region that develops at the lower end of the laminar recirculating flow region moves closer to the step (see Fig. 14). They also found that in the laminar non-circulating flow region downstream of the step, the local Nusselt number decreases with increasing buoyancy-opposing force and the reverse trend may occur inside the recirculating flow region (see Fig. 15). This finding is opposite of the trend observed for the buoyancy-assisting case.

Table 4
Laminar mixed convection flow over forward-facing step

	As the parameter increases		
	Step height	Velocity	Buoyancy-assisting
Local heat transfer rate	Decreases	Increases	Increases
Reattachment length, x_r	Increases	Increases	Decreases

In summary, the effects of step height, velocity, buoyancy due to downstream wall heating on the reattachment length and local Nusselt number of laminar mixed convection over a forward-facing step are tabulated in Table 4.

3. Concluding remarks and recommendations

This paper has presented a comprehensive review of the research on single-phase laminar mixed convection flow over backward- and forward-facing steps. A detailed summary of the effects of several parameters such as step height, Reynolds number, expansion ratio, inclination angle, Prandtl number, and buoyancy-force (assisting and opposing) on the flow and thermal fields downstream of the step has been presented. Correlation equations that predict the reattachment lengths of the recirculation regions that may develop upstream and/or downstream of the step are summarized and presented in tabular form for quick reference.

This review clearly shows that laminar mixed convection flow over a forward-facing step has received very little attention comparing to that of the backward-facing step case. This is may be because in the forward-facing step geometry, two recirculation regions can develop, which make it more complicated to study than the backward-facing step geometry in which only one separated region occurs behind the step. Further work is needed in the area of laminar mixed convection flow over forward-facing step. This work includes, but is not limited to, the effects of inclination angle and Prandtl number on the flow and thermal fields characteristics in this step geometry, and uniform wall heat flux case. In addition, it would be of great value that a benchmark heat transfer problem involving the laminar mixed convection flow over a forward-facing step be organized to check the capabilities of the numerous computer codes that are available.

It is clear from this review that a very limited number of three-dimensional studies that has investigated laminar mixed convection flow over backward-facing step. Further three-dimensional simulations of laminar mixed convection flow over both backward- and forward-facing steps is needed.

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